

MATH 365 Homework #12

Given a matrix A as follows

$$A = \begin{pmatrix} 58.6601 & 0.8355 & 2.8715 & 2.3766 & 2.0362 & 2.1181 \\ 2.7174 & 39.3755 & 1.4561 & 2.8785 & 2.2732 & 0.0955 \\ 0.381 & 2.8725 & 57.6202 & 1.9672 & 2.2294 & 0.8308 \\ 2.7401 & 2.8947 & 0.4257 & 2.5712 & 1.1767 & 0.1385 \\ 1.8971 & 0.4728 & 1.2653 & 2.5474 & 47.1944 & 0.2914 \\ 0.2926 & 2.9118 & 2.7472 & 2.802 & 0.5136 & 59.289 \end{pmatrix}.$$

1. (10 points) Based on the “Power Method”, **write and run a real source code on the computer** to find the largest eigenvalue (in modulus) of the above matrix A and the corresponding eigenvector. Show the iteration information at each step, i.e. the step number, the iteration error, the largest eigenvalue and corresponding eigenvector.
2. (20 points) Based on the “Inverse Power Method”, **write and run a real source code on the computer** to find the smallest eigenvalue (in modulus) of the above matrix A and the corresponding eigenvector. Show the iteration information at each step, i.e. the step number, the iteration error, the smallest eigenvalue and corresponding eigenvector.
3. (10 points) Find the condition number of the above matrix A . Determine if this matrix is well-conditioned or ill-conditioned.
4. (20 points) Find the spectral radius of the iteration matrix of Jacobi iterative method for the linear system $Ax = b$. Determine if Jacobi method is convergent or divergent for this linear system $Ax = b$, where b can be any given right-hand-side vector.
5. (20 points) Find the spectral radius of the iteration matrix of Gauss-Seidel iterative method for the linear system $Ax = b$. Determine if Gauss-Seidel method is convergent or divergent for this linear system $Ax = b$, where b can be any given right-hand-side vector.
6. (20 points) Find the spectral radius of the iteration matrix of SOR iterative method for the linear system $Ax = b$, and determine the optimal ω for a convergent SOR method, where b can be any given right-hand-side vector.

Note: In your homework, the right source code and the correct output results must be shown altogether in order to obtain full score.