# MATH 365 Homework #3

Due: Wednesday, Feb. 8, Spring 2012

## Review of "Determinant Expansion by Minors"

***1. Principle***

Let $|A|$ denote the determinant of a square matrix $A \in \mathbb{R}^{n\times n}$, then

$$|A| = \det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{i,j} M_{i,j}$$

where $M_{i,j}$ is called a minor of $A$ with respect to $a_{i,j}$, defined by the determinant of a smaller matrix $A_{i,j} \in \mathbb{R}^{(n-1)\times(n-1)}$ which is obtained by crossing out row $i$ and column $j$ from matrix $A$, i.e., $M_{i,j} = \det(A_{i,j})$.

Alternatively, the factor $(-1)^{i+j}$ is sometimes absorbed into the minor and the formula of **determinant expansion by minors** can be rewritten as

$$|A| = \det(A) = \sum_{j=1}^{n} a_{i,j} C_{i,j}$$

where $C_{i,j}$ is called a **cofactor** and defined as $C_{i,j} = (-1)^{i+j} M_{i,j}$.

***2. Pseudo-code***

We shall use **recursive** method to implement **determinant expansion by minors**.

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`Pseudo-code of determinant expansion by minors`

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Function determinant(A, n)
if n = 2 then return a[1][1]*a[2][2] - a[1][2]*a[2][1]
det := 0
for i := 1 to n
{
  Minor[x][y] = A[x][y] where x != 1 & y != i
  det := det + (-1)^(1+i)*a[1][i]*determinant(Minor[1][i],n-1)
}
return det
```

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1. Given the following matrices

$$A = \begin{pmatrix} 0.6791 & 0.0987 & 0.4942 & 0.0305 & 0.8055 & 0.9787 & 0.5216 & 0.973 \\ 0.3955 & 0.2619 & 0.7791 & 0.7441 & 0.5767 & 0.7127 & 0.0967 & 0.649 \\ 0.3674 & 0.3354 & 0.715 & 0.5 & 0.1829 & 0.5005 & 0.8181 & 0.8003 \\ 0.988 & 0.6797 & 0.9037 & 0.4799 & 0.2399 & 0.4711 & 0.8175 & 0.4538 \\ 0.0377 & 0.1366 & 0.8909 & 0.9047 & 0.8865 & 0.0596 & 0.7224 & 0.4324 \\ 0.8852 & 0.7212 & 0.3342 & 0.6099 & 0.0287 & 0.682 & 0.1499 & 0.8253 \\ 0.9133 & 0.1068 & 0.6987 & 0.6177 & 0.4899 & 0.0424 & 0.6596 & 0.0835 \\ 0.7962 & 0.6538 & 0.1978 & 0.8594 & 0.1679 & 0.0714 & 0.5186 & 0.1332 \end{pmatrix},$$

$$B = \begin{pmatrix} 2.0373 & 0.2961 & 1.4826 & 0.0915 & 2.4165 & 2.9361 & 1.5648 & 2.919 \\ 1.1865 & 0.7857 & 2.3373 & 2.2323 & 1.7301 & 2.1381 & 0.2901 & 1.947 \\ 1.1022 & 1.0062 & 2.145 & 1.5 & 0.5487 & 1.5015 & 2.4543 & 2.4009 \\ 2.964 & 2.0391 & 2.7111 & 1.4397 & 0.7197 & 1.4133 & 2.4525 & 1.3614 \\ 0.1131 & 0.4098 & 2.6727 & 2.7141 & 2.6595 & 0.1788 & 2.1672 & 1.2972 \\ 2.6556 & 2.1636 & 1.0026 & 1.8297 & 0.0861 & 2.046 & 0.4497 & 2.4759 \\ 2.7399 & 0.3204 & 2.0961 & 1.8531 & 1.4697 & 0.1272 & 1.9788 & 0.2505 \\ 2.3886 & 1.9614 & 0.5934 & 2.5782 & 0.5037 & 0.2142 & 1.5558 & 0.3996 \end{pmatrix},$$

$$C = \begin{pmatrix} 0.5202 & 1.9706 & 1.1172 & 0.8074 & 1.999 & 0.0978 & 0.4692 & 1.7685 \\ 0 & 1.8839 & 0.5944 & 1.2685 & 1.6174 & 1.6836 & 2.5666 & 0.6786 \\ 0 & 0 & 1.4691 & 1.6436 & 2.0943 & 2.6456 & 1.9343 & 1.1539 \\ 0 & 0 & 0 & 2.8282 & 1.9996 & 2.0075 & 1.1288 & 1.749 \\ 0 & 0 & 0 & 0 & 0.5344 & 0.5713 & 0.5728 & 0.7554 \\ 0 & 0 & 0 & 0 & 0 & 1.1067 & 1.2848 & 0.8713 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.4461 & 1.8513 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7958 \end{pmatrix},$$

based on the pseudo-code of "Determinant Expansion by Minors", **write and run a source code on the computer** to

(i) (10 points) compute $\det(A)$ and $\det(B)$.

(ii) (20 points) verify whether $\det(AB)=\det(A)\det(B)=\det(BA)$.

(iii) (10 points) verify whether $\det(A^t)=\det(A)$.

(iv) (10 points) compute $\det(C)$ and verify whether $\det(C)=\prod_{i=1}^{8} c_{i,i} = c_{1,1}c_{2,2}\cdots c_{8,8}$.

(v) (20 points) compute $A^{-1}, B^{-1}$ and $C^{-1}$.

(vi) (10 points) verify whether $(AB)^{-1} = B^{-1}A^{-1}$.

(vii) (10 points) Compute $tr\ A$ and $tr\ B$, where $tr$ means the trace of matrix.

(viii) (10 points) verify whether $tr(AB) = tr(BA)$.

**Note: In your homework, the right source code and the correct output results must be shown altogether in order to obtain full score.**