

MATH 767 Test #1

February 9, 2011

Name: _____ Student ID: _____

DUE ON: FEB. 14, 2011

THE USE OF BOOKS, NOTES ETC. IS ALLOWED.

Given a one-dimensional boundary value model problem as follows

$$\begin{aligned} -u''(x) &= 0, & 0 < x < 1, \\ u(0) = u(1) &= 0, \end{aligned} \tag{1}$$

use **finite difference method (FDM)** and the **weighted Jacobi** method to solve the above model problem with $\omega = \frac{2}{3}$ and the initial guess consisting of the following modes:

1. w_2 : the mode with $k=2$.
2. w_{16} : the mode with $k=16$.
3. $(w_2 + w_{16})/2$: two modes with $k=2$ and 16.

What you shall deliver are the following:

1. (15 points) FDM discretization, including grid and scheme.
2. (15 points) The weighted Jacobi iterative scheme.
3. (5 points) Write and plot the required three initial guesses consisting of above Fourier modes.
4. (40 points) Write, debug and run an elaborate **source code** in any computer language (Matlab, C, Fortran, etc.) to implement above numerical schemes, and obtain the numerical solutions v for each initial guess.
5. (10 points) Plot the numerical solutions v after 1 iteration, 10 iterations and 100 iterations for each initial guess. To that end, you need to choose a sufficiently small tolerance for the stopping criteria.

6. (10 points) Calculate the **relative approximation error** in the maximum norm and its logarithmic representation, and plot these two errors against the number of iterations.
7. (5 points) Conclusion remark based on your above numerical experiments.